VOLUME II-2

ESTIMATED LONG-TERM MEAN CONCENTRATION OF EFFLUENT FROM SONGS

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ESTIMATED LONG-TERM MEAN CONCENTRATION OF EFFLUENT FROM SONGS

The quantity that is estimated in the following calculations is the long-term mean concentration of a hypothetical conservative tracer discharged from SONGS, relative to its original concentration in the discharged water. This quantity cannot be observed at ranges of several kilometers or more except by long-term recordings of some highly detectable tracer always present in the discharge. Since we do not have these, the best we can do is to estimate the long-term mean concentration theoretically from available long-term statistics of the local currents. The tracer is taken to be released continuously at a uniform rate, regardless of its concentration in the water taken in by SONGS, so the results do not apply directly to effluents for which the release-rate depends on the intake concentration.

1) Premises:

The conceptual experiment on which these calculations are based is the instantaneous release of unit mass of dye at the point \( x_i = 0 \) in a turbulent medium with velocity-field \( V_i \), resulting in a concentration \( \rho \) at any point \( x_i \) and time \( t \) after the release. The dye-release is repeated at many random instants over so long a span of time that the statistics of \( V_i \) can be taken as stationary. It is postulated that the ensemble-mean concentration \( \langle \rho \rangle \) taken over this set of releases has a normal distribution in three dimensions that can be written on principal axes as

\[
\langle \rho \rangle = \prod_i (2\pi)^{-1/2} \sigma_i^{-1} \exp\left\{ -\frac{(x_i - \mu_i)^2}{2 \sigma_i^2} \right\}.
\]

This expression is a solution of the advection-dispersion equation \( \partial \langle \rho \rangle / \partial t = -\partial \mu_i / \partial t \cdot \partial \langle \rho \rangle / \partial x_i + K_i \partial^2 \langle \rho \rangle / \partial x_i^2 \), with \( 2K_i = \partial \sigma_i^2 / \partial t \). This may be transformed to the isotropic form in which the last term is replaced by \( K \nabla^2 \langle \rho \rangle \), with \( K = (K_x K_y K_z)^{1/3} \), by replacing each coordinate \( x_i \) with \( (K_i / K)^{1/2} x_i \) and each velocity \( V_i \) with \( (K_i / K)^{1/2} V_i \).
A mathematical identity due to G.I. Taylor states that

\[ 2K_i = \frac{\partial \bar{\sigma}_i^2}{\partial t} = 2 \int_0^t \hat{C}_i(\tau) d\tau, \]

in which \( \hat{C}_i(\tau) \) is the autocovariance function of \( V_i \) for lag \( \tau \), formed by averaging lagged products of particle-velocities \( V_i \) over an ensemble of particles released from the same point and subtracting the square of ensemble-averaged \( V_i \). Here the time-variable \( t \) represents the time since release and not absolute time, so this ensemble can be formed from a succession of releases as above. For this particular ensemble we can also say \( \mathcal{M}_i = \langle V_i \rangle t \), since \( \langle V_i \rangle \) is a constant stationary mean.

It is also postulated that the velocity-field \( V_i \) is both stationary and homogeneous (that is, ergodic), so that the statistics \( \langle V_i \rangle \) and \( \hat{C}_i(\tau) \) given by ensemble-means can be replaced by the corresponding mean velocities \( \bar{V}_i \) and autocovariance functions \( C_i(\tau) \) formed by time-averaging at a fixed point. On this postulate, \( \langle V_i \rangle \) and \( K_i \) can be found from the records of fixed current-meters.

The time-averaged \( C_i(\tau) \) is the variance times the autocorrelation function, i.e. \( C_i(\tau) = \langle V_i'^2 \rangle R_i(\tau) \). As \( t \) approaches 0, \( \frac{\partial \bar{\sigma}_i^2}{\partial t} \) approaches \( 2 \langle V_i'^2 \rangle t \), so that \( \bar{\sigma}_i^2 \) approaches \( \langle V_i'^2 \rangle t^2 \), which is the upper limit for \( \bar{\sigma}_i^2 \). At large \( t \), \( \frac{\partial \bar{\sigma}_i^2}{\partial t} \) approaches the constant value \( 2 \langle V_i'^2 \rangle i \), in which \( i \) is the integral of \( R_i(\tau) \) from zero to infinity. As approximations to the integrals of actual autocovariance functions, we will use the form \( K_i = \sqrt{v_i^2} t \) for \( t \) between zero and a chosen time \( T \), and \( K_i = \sqrt{v_i^2} T \) thereafter; here \( \sqrt{v_i^2} \) is some fraction of \( \sqrt{v_i^2} \). Integration of these gives \( \bar{\sigma}_i^2 = \sqrt{v_i^2} t^2 \) for times in the interval \( 0 < t < T \), and \( \bar{\sigma}_i^2 = 2 \sqrt{v_i^2} T(t - T/2) \) in \( t > T \).

2) Superposition of point-releases.

To simplify the notation, the integrations that follow will be carried out as if the medium were isotropic, and the final results will be transformed back to the actual medium with different \( K \)'s for the longshore and cross-shelf directions.

In cylindrical coordinates \( r \), \( \theta \), and \( z \), with the \( z \)-axis on the shoreline and \( \theta = 0 \) at the sea surface, the nearshore waters are taken to be bounded by the two planes \( \theta = 0 \) and \( \theta = \beta \). (The slope of the bottom \( \beta \) is a small
angle, and we will not distinguish between the angle and its sine or tangent.) By symmetry, material released from a ring-shaped source at \( r = a \) in an unbounded medium will not cross either plane, so we represent the actual source, of total strength \( Q \rho_0 \), in the sector \( 0 < \theta < \beta \) between the boundaries, as a ring-shaped source at \( r = a \), \( z = 0 \), in unbounded space, with strength \( Q \rho_0 / \beta \) per radian. This approximation gives concentration uniform with \( \theta \) in the wedge.

In these coordinates, the normal distribution of concentration due to release of mass \( Q \rho_0 d\theta dt / \beta \) from the ring-element \( d\theta \) in the time-element \( dt \) is written as

\[
d\rho = [Q \rho_0 d\theta dt / (2\pi)^{3/2} \beta] \exp\{-[(R - \hat{\mu})^2 / 2 \sigma^2]\},
\]

in which \( \mu_2 = \mu t \), the mean longshore velocity times the time, and \( (R - \hat{\mu})^2 = r^2 + a^2 - 2r \cos \theta + z^2 - 2wz + w_z t^2 \). Integrating this equation over the circle \( -\pi < \theta < \pi \) gives the long-term mean or expected value of concentration due to an instantaneous release of mass \( Q \rho_0 dt \) from the segment of ring-source in the wedge-shaped space. A further integration over the range of age \( t' < t < \infty \) gives the long-term mean concentration due to a continuous release of tracer at the rate \( Q \rho_0 \) from indefinitely long ago up to time \( t' \) before the present.

3) The case \( w = 0 \):

We have not managed to integrate (3) in general, but we can describe the overall situation fairly well by treating the special case of no mean longshore current \((w = 0)\) and later considering the case \( r = 0 \) that gives the concentration on the shoreline for any value of \( w \).

With \( w = 0 \), we will first examine the effect of the change of \( K \) from \( \sqrt{2} t \) to \( \sqrt{2} T \) at \( t = T \) in the approximation we have adopted. The integration over time from \( T \) to infinity with \( K = \sqrt{2} t \) throughout gives

\[
d\rho = [Q \rho_0 d\theta / (2\pi)^{1/2} \beta] \exp\{-[R^2 / 2 \sigma^2 T^2]\} \cdot \exp\{-[-R^2 / 2 \sigma^2 T^2]\},
\]

with \( R^2 = r^2 + a^2 - 2r \cos \theta + z^2 \).

As \( T \to 0 \), this reduces to

\[
\rho_0 \cdot [Q \rho_0 d\theta / (2\pi)^{1/2} \beta] \cdot \exp\{-[R^2 / 2 \sigma^2]\}.
\]
Now taking $\sigma^2 = j^2 T^2$ at $T$ and $K = j^2 T$ thereafter, we get

$$\rho' = \frac{(Q \rho_0 / 4\pi \beta j^2 RT) \text{erf}(R^2 / 2j^2 T^2)^{1/2}}{2j^2 T^2}.$$ 

Here it is useful to introduce the quantities $D^2 = (r-a)^2 + z^2$ and $D'^2 = (r+a)^2 + z^2$. $D$ is the distance from the source at $(a,0,0)$ and is the minimum value of $R$ over $\theta$. $D'$ is the distance from the inland image-point of the source at $(a,0,\pi)$ and is the maximum of $R$ over $\theta$.

At points where $D'^2 / 2j^2 T^2$ is less than 1/4 or so, $\rho'$ approaches $R^2 / 2j^2 T^2$ times $\rho_0$, and $\rho'\tau'$ approaches twice that value. The relative error that arises by taking $K = j^2 t$ at all times instead of stopping its growth at $t = T$ is then given by $R^2 / 2j^2 T^2$ in regions where the image-distance $D'$ is half of $2^{1/2}j T$ or less.

If $T$ is set equal to zero in equation $(4)$, $\rho'$ has a singularity at the source $(a,0,0)$ which is entirely due to the most recent idealized point-release in the time-element $dt$ just before the present. Any actual release will have some initial extent in space, which can be most easily represented by $\sigma_0 = \sqrt{t_1}$, giving a fictive age of $t_1$ at birth to the release in each time-element. Substituting $t_1$ for $T$ in $(4)$ and setting $r = a$ and $z = 0$, we can integrate over $\theta$ to find the resulting concentration $\rho_1$ at the source. The result is

$$\rho_1 / \rho_0 = [Q \exp(-b) / (2\pi)^{1/2} \beta] \cdot 2j^2 t_1^2 [I_0(b) + I_1(b)],$$

in which $I_0$ and $I_1$ are modified Bessel functions and

$$b = a^2 / j^2 t_1^2.$$ For $b$ greater than 3 or so, this equation approaches

$$(5) \quad \rho_1 / \rho_0 = Q / 2\pi \beta j^2 a t_1 = Q / 2\pi \beta j a \sigma_0.$$ 

This expression allows the fictive age $t_1$ or the initial dispersion $\sigma_0$ to be assigned from some estimate of the initial dilution $\rho_1 / \rho_0$ in the neighborhood of the source.

In the outer region where $D^2 / 2\sigma_0^2$ is greater than about 3, equation $(4)$ is well approximated by setting the exponential term to zero, which shows that $\rho'$ in this region is independent of the initial dispersion $\sigma_0$. For this region, we will integrate over $\theta$ with $t_1$ set to zero. The result is
At this point we will transform back to the actual coordinates with anisotropic, \( K_i = \sqrt{1^2} t \), by substituting \( (\sqrt{r} \sqrt{z})^{1/3} \) for \( \sqrt{r} \sqrt{z} \) for \( r \), and \( \sqrt{z} \) for \( z \). The unknown vertical parameter \( \sqrt{d} \) drops out in this process. We will also scale the dimensions relative to the distance \( a \) of the source from the shore, with the notation \( r' = r/a \) and \( z' = z/a \). In this form, the concentration in the absence of current is given by

\[ \frac{P}{P_0} = \frac{Q}{(2\pi)^{1/2} \beta} \int DD' \, \sqrt{z} a^2 \, G \right) \]

The first factor in square brackets, which we will call \( B \), gives the concentration at the point \( r = 0 \), \( z = 0 \), (on the shoreline closest to the source) where \( G = 1 \), and \( G \) gives the concentration at other points relative to this value. Figure 1 shows contours of \( G \) on a plot of the scaled coordinates \( r' \) and \( qz' \).

4) The case \( r = 0 \), with \( K = \sqrt{2} t \) at all times:

The other special case we can deal with is to evaluate \( P \) on the shoreline \( r = 0 \) in the presence of a mean longshore current \( w \). For this case, the integration over \(-\pi < \theta < \pi\) and over \( 0 < t < \infty \) gives

\[ \frac{P}{P_0} = \frac{Q}{(2\pi)^{1/2} \beta} \sqrt{z} a^2 \, G_0 \int F(pqz'/(1+q^2z'^2)^{1/2}) \right) \]

in which

\[ G_0 = (1+q^2z'^2)^{-1} , \quad p = w/2^{1/2} \sqrt{z} \right) \]

and the function

\[ F(s) = 1 + \pi^{1/2} \cdot s \cdot \exp(s^2) \cdot (1+\text{erf}(s)) \right) \]

The first factor is \( B \), as in \( 7 \); the spatial factor \( G_0 \) is simply \( G \) evaluated at \( r = 0 \); the factor \( \exp\{-p^2\} \) gives
a uniform decrease of \( \rho \) everywhere with increasing mean current speed; lastly, the directional factor \( F \) increases \( \rho \) at places downcurrent from the source (positive \( z \)) and decreases \( \rho \) at upcurrent places. Figure 2 shows plots of the quantity \( H = G_0 \exp\{-p^2\}F\left[1+(q^2z'^2)^{1/2}\right] \) against the scaled longshore distance \( qz' \) for different values of \( p \). This is the concentration relative to its value at \( r = 0, z = 0 \), which is given as before by the leading factor \( B = Q/(2\pi)^{1/2}p^2\sqrt{z}a^2 \).

5) The conditions \[ D'/2 \sqrt{2T^2} < 1/4, \quad a^2/\sqrt{2t_1^2} > 3 \]
and \[ D^2/2\sqrt{2t_1^2} > 3. \]

In 3) above it was shown that the relative error due to taking \( K = \sqrt{2t} \) at all times instead of taking \( K = \sqrt{2T} \) for times greater than a chosen \( T \) is less than \( D'/2\sqrt{2T^2} \) at places where that quantity is less than about \( 1/4 \). In the actual scaled coordinates this region is defined by

\[ [(r+a)^2 + q^2z^2]/2q^2\sqrt{2T^2} < 1/4. \]

Then it was shown in 4) that the initial dispersion \( \mathcal{G}_0 = \sqrt{v_1} \), which determines concentration close to the source, does not materially affect concentration in the outer region given by \( D^2/2\mathcal{G}_0^2 > 3 \). The initial dispersion corresponding to a given nearfield concentration \( \rho_1 \) was given approximately as \( \mathcal{G}_0 = Q(\rho_0/\rho_1)/2\pi\beta \sqrt{v}a \) for \( a^2/\mathcal{G}_0^2 > 3 \). In actual scaled coordinates, the region in which equations \{7\} and \{8\} apply without regard to the initial dispersion is defined by

\[ [(r-a)^2 + q^2z^2]/2q^2\sqrt{2t_1^2} > 3, \]

and the fictive age \( t_1 \) is given as

\[ t_1 = Q(\rho_0/\rho_1)/2\pi\beta q\sqrt{v}z^2a, \] if \( a^2/q^2\sqrt{2t_1^2} > 3 \).

6) Estimates of parameters:

With equation \{7\} and Figure 1 for the case \( w = 0 \), and equation \{8\} and Figure 2 for the case \( r = 0 \), we can now estimate long-term mean relative concentrations \( \rho/\rho_0 \) due to a continuous discharge into a wedge-shaped space, given the discharge-rate \( Q \), the included angle \( \beta \), the distance \( a \) of the source from the shoreline, the longshore and cross-shelf
dispersion parameters $V_z$ and $V_r$, and the long-term mean longshore current velocity $w$. To estimate the range of distances to which these estimates apply, from equations (9), (10), and (11), we also need values for $T$, the age at which the actual dispersion parameters approach constant values, and the initial dilution $\rho_0/\rho_1$.

The parameters $Q$, $\beta$, $V_z$, and $a$ appear together in the combination $B = Q/(2\pi)^{1/2}a^2$, which gives $\rho/\rho_0$ at the point $r = 0$, $z = 0$, on the shoreline nearest the source, in the absence of current. The source-distance $a$ also appears as a scale-factor: doubling the value of $a$ doubles the actual distance represented by a scale distance such as $r' = r/a$, besides reducing the concentration at a given scale-distance by a factor of four. The parameter $V_r$ appears in the ratio $q = V_r/V_z$, and the mean velocity $w$ appears in the ratio $p = w/(2^{1/2}V_z)$.

The discharge rate $Q$ is well-determined by the pumping-rate: in round numbers, $10^6$ cm$^3$/sec for SONGS Units 2 and 3 together, and $2 \times 10^7$ cm$^3$/sec for Unit 1.

The slope $\beta$ can be taken as the actual mean bottom-slope, which is fairly well approximated by $6 \times 10^{-3}$ out to about 5 km from shore. In that case, $\rho/\rho_0$ will represent the top-to-bottom mean of relative concentration, without regard to how deeply the actual discharge is mixed. If $\rho/\rho_0$ were taken instead as the relative concentration within a discharge plume restricted to water above a thermocline, $\beta$ might well be taken as small as $3 \times 10^{-3}$. The actual mean isotherms do slope downward away from the shore, so the wedge-shaped space is still a suitable idealization for this case.

The source-distance $a$ is well-defined for Unit 1 as about 800m or $8 \times 10^4$ cm from shore. For Units 2 and 3 together, $a$ is ill-defined, since the diffusers extend from about 1 to 2.5 km offshore and the discharge has considerable initial offshore momentum. In weak currents, the discharged water appears to form a pool beyond the end of the outer diffuser and to spread and disperse from there; as current increases, the discharged water appears to move less far offshore before losing its initial momentum. As a very rough average over all conditions, we will take the effective source-distance as 2.5 km or $2.5 \times 10^5$ cm for Units 2 and 3 together, acknowledging that this is an uncertain estimate.
The initial dilution \( \rho_0/\rho_1 \) is fairly well estimated by scale-model and field experiments as about 8 at 100m or so away from the diffusers, and about 20 at 1 km.

The other parameters \( w, v_z, v_r, \) and \( T \) are estimated from long current records in the vicinity. The records we have used are those of hourly velocities at 3m depth over the years 1979-80 and 1984-6 (the records of intervening years were not stationary). To minimize data-gaps, we formed composite records from different locations in 10 to 15m total depth of water, far enough from the diffusers to avoid gross distortion of the natural velocity-field by SONGS. The six-year mean longshore velocity \( \bar{v}_z \) or \( w \) is 2.9 cm/sec, directed downcoast. (The mean cross-shelf velocity is 0.1 cm/sec onshore, much less than the instrumental zero-uncertainty). The six-year standard deviation of longshore velocity \( (\bar{v}_z)^{1/2} \) is 10.2 cm/sec and that of cross-shelf velocity \( (v_r)^{1/2} \) is 4.0 cm/sec.

The autocorrelation functions \( R(\tau) \) of longshore and cross-shelf velocities in the two stationary sets of years are all similar enough to be represented by a single composite function without material error. This function was graphically smoothed (to remove tidal periodicities) and integrated over \( \tau \), with the result shown in Figure 3. The curve in the Figure is that of \( \int_0^\tau R(\tau) \, d\tau = K_i/(\bar{v}_i^{1/2}) \). The asymptote \( I \) of the integral is about 36 hours. The straight line through the origin corresponds to the form \( K_i = \bar{v}_i^{1/2} t \) that we have chosen to approximate \( K_i \) for times less than a chosen \( T \). This line has the slope \( \bar{v}_i^{1/2}/(\bar{v}_i^{1/2}) = 1/2 \), and reaches the asymptote \( I \) at \( T = 21 \). With the observed standard deviations of velocity, the chosen line gives the estimates \( \bar{v}_z = 7.2 \) cm/sec, \( \bar{v}_r = 2.8 \) cm/sec, and \( T = 72 \) hours = \( 2.6 \times 10^8 \) sec.
A particular set of parameters chosen for calculations on Units 2 and 3 together is tabulated below:

\[ Q = 1 \times 10^8 \text{ cm}^3/\text{sec} \quad ; \quad a = 2.5 \times 10^5 \text{ cm} \]

\[ \beta = 6 \times 10^{-3} \quad ; \quad \rho_0/\rho_1 = 12 \]

\[ \mathcal{V}_z = 7.2 \text{ cm/sec} \quad ; \quad \mathcal{V}_r = 2.8 \text{ cm/sec} \]

\[ w = 2.9 \text{ cm/sec downcoast} \quad ; \quad T = 2.6 \times 10^5 \text{ sec} \]

\[ q = \mathcal{V}_r/\mathcal{V}_z = 0.39 \quad ; \quad p = w/2^{1/2}\mathcal{V}_z = 0.28 \]

The combination \( B = Q/(2\pi)^{1/2}/\beta \mathcal{V}_z a^2 \) with these values comes out to be \( 0.015 \). This is the estimate of long-term mean relative concentration \( \rho/\rho_0 \) at the shoreline point \( r = 0, z = 0 \), in the absence of current.

7) Regions of applicability:

Before looking at the spatial distribution of \( \rho/\rho_0 \) or the effect of current, we will consider the outer and inner limits of the range of distance in which these calculations apply. Putting the relevant values in equation (9), we get \( [(r + 2.5 \text{km})^2 + 0.15z^2]/106 \text{km}^2 < 1/4 \) as the region in which we can disregard the levelling-off of \( K_1 \) to constant values at large times. Inside this region, this ratio itself is the maximum fractional error: on the shoreline at \( r = 0 \), the error is 20% or less at \( z = 10 \text{km} \), and 8% or less at \( z = 4 \text{km} \); at \( r = a \), it is at most 25% at \( z = 2 \text{km} \).

With the tabulated parameters, equation (11) gives \( t_1 = 6300 \text{ sec} \), or 1.75 hours, corresponding to an initial dispersion \( \mathcal{G}_0 \) on the order of 300m. The ratio \( a/q\mathcal{V}_zt_1 \) comes out to be about 14, so the condition on (11) is fully met and the expression for \( t_1 \) is a close approximation.

With the same parameters, the region defined by equation (10) becomes \( [(r-2.5 \text{km})^2 + 0.15z^2]/0.063 \text{km}^2 > 3 \). For any value of \( r \), the widest bounds for this region can formally be set at \( z^2 = 1.3 \text{ km} \); at greater longshore distances the
initial dilution of the discharge will not significantly affect the long-term mean concentrations calculated from (7) and (8). These bounds should not be taken very literally, though, since the representation of initial dispersion by $\varphi_0$ is an artifice that takes no account of the actual characteristics of the diffusers except for the observed nearfield dilution.

8) Estimated long-term mean relative concentration:

At this point, the estimated long-term mean relative concentration $\rho/\rho_0$ for the given parameters may be portrayed simply by re-labeling the scaled axes of Figures 1 and 2 so that $qz/a = 1$ corresponds to 6.4 km and $r/a = 1$ corresponds to 2.5 km, and then multiplying the plotted functions $G$ and $H$ by the factor $B = .015$. To show $\rho/\rho_0$ in the absence of current on an undistorted map, Figure 4 is redrawn from Figure 1 with equal scales of actual longshore and offshore distance. On the seaward side of the points where the contours of $\rho/\rho_0$ on this Figure are broken and labelled, the relative error due to disregarding the levelling-off of $K$ with time may exceed 25%, as discussed in 8) above. The errors are probably smaller than this within 6 or 7 km from the source, though, so dashed contours have been drawn in the seaward region to show generally how the concentration falls off with distance offshore from the source. The .02 contour is not shown inshore of the source where it might be seriously in error because of the initial dispersion.

Figure 5 corresponds to Figure 2 and shows $\rho/\rho_0$ at the shoreline for no current ($w = 0$) and for the actual long-term mean current $w = 2.9$ cm/sec downcoast ($p = .28$). The principal effect of the actual current is to displace the pattern of concentration downstream by about 1.9 km, without materially changing its size or shape. This will also be true at offshore points not too close to the source, so we have shown the expected effect of the actual current in Figure 6 by displacing the contours in Figure 4 a distance of 1.9 km downcoast. Because of the uncertain effect of current close to the source, the .02 contour is not shown.

Figure 6 shows the pattern for the actual long-term mean current of 2.9 cm/sec, but the pattern for no current shown in Figure 4 is not entirely hypothetical. Seasonal means of the current may vary from close to zero in some winters to about 6 cm/sec or more downcoast in some summers, without much
change in the variances of longshore or cross-shelf velocity. While Figure 6 represents the overall mean, Figure 4 might represent the mean for a winter of no mean current, and a plot similar to Figure 4 with the contours displaced about 4 km downcoast might represent the mean for a summer with mean current 6 cm/sec downcoast.

9) Uncertainties:

Finally we should examine the postulates and chosen values of parameters that went into these calculations, to say what we can about the uncertainty of the results.

The beginning postulate of a normal joint distribution for displacements is necessary if we are to proceed at all, but it has no firm physical basis, and may at worst have to fall back on the Central Limit Theorem. This postulate cannot be checked at a particular place except through a large set of drogue experiments to observe the distribution of displacements directly. The postulates that the velocity-field is stationary and homogeneous may be checked from the maps of principal current components and the autocovariance functions shown in other chapters; in the region we are concerned with these postulates are neither strictly true nor grossly unrealistic.

For these calculations the most serious effect of a non-ergodic field is probably to make the particle-averaged autocovariance function \( \hat{C}(\tau) \) different from the time-averaged \( C(\tau) \) at a fixed point. In the CODE experiment off Northern California (Davis, 1985), longshore \( \hat{C} \) fell off to zero in about 3 days, while \( C \) took nearly 6 days, as at San Onofre.

If we were fitting lines to the integral of particle-averaged \( \hat{R} \) in Figure 3, instead of to the integral of time-averaged \( R \), we might choose a somewhat different \( \tau \), giving a different slope to the chosen line \( K_1 = \hat{V}_1^2 t/(\hat{v}_1'^2) \). The difference is unlikely to be major, though, since both integrals have the same initial slope of 1. As shown in 3) above, the concentration within a certain distance is mainly influenced by the early history of \( \hat{K} \), that is, by the variance of the velocity, so if the particle-averaged and time-averaged variances are about the same the results in this range will not be much affected by uncertainty in the autocorrelation function.
Representing the diffusers of Units 2 and 3 by a nominal point-source at a distance $a$ from shore gives a considerable uncertainty at close range, falling off with distance. The allowance for initial dispersion gives some reassurance that the resulting error will be minor at distances of 2 or 3 km, except for the uncertainty in assigning the distance $a$. We have chosen the value of 2.5 km for $a$, but if we knew more we might well assign a somewhat different value. If the actual best value were 2 km, say, the values of $\rho/\rho_o$ in Figures 4 through 6 would be multiplied by 1.56 and the distances on the axes would be multiplied by 1.25.

The rest of the parameters are comparatively well-defined, though none of them are precise. We do not know enough to set objective confidence limits on the final results, but we believe the the plotted values of $\rho/\rho_o$ at given distances are good to a factor of about three or four. By way of confirming the order of magnitude, we note that the independent observations of dilution in the plume at about 1 km from the diffusers show relative concentrations on the order of .05 in about 1/3 of the water column. If this occurs about half the time at a given place, the resulting long-term mean would be on the order of .01, in a region where the calculations (not modified by allowance for initial dispersion) would predict something like .03. These two numbers need not agree closely, but it is reassuring that they are not an order of magnitude apart.

Reference:

Figure 1. Contours of $G$ on $r/a$ and $qz/a$ (see p. 5).
Figure 2. Plot of $H$ vs. $qz/a$ for various $p$ (see p. 6).
Figure 5. Plot of $I \cdot R dr$ vs. $t$ (see p. 8).

Figure 3. Plot of $\int_0^t R dr$ vs. $t$ (see p. 8).
Figure 4. Contours of relative concentration $\rho/\rho_0$ with no mean current (see p. 10).
Figure 5. Plot of $\rho/\rho_o$ at the shoreline for mean currents of 0 and 2.9 cm/sec (see p. 10).
Figure 6. Contours of relative concentration \( \rho/\rho_o \) for mean current 2.9 cm/sec downcoast (see p. 10).